HIGH-SCHOOL MATHEMATICS

THIRMUTHIS BROOKMAN
High School, Berkeley, Cal.

For some years Discipulus had taught mathematics with increasing dissatisfaction. For some years he had initiated high-school freshmen into the mysteries of elementary algebra, yet found that less than one-fifth continued the subject in their third year and still fewer made use of it in college. The numbers of entering freshmen however never decreased. All were put through their mathematical paces for the sake of the magic fifth who studied the advanced work. His best efforts failed to find an answer to the question: *Does the one year's training in algebra justify itself for those who do not continue the subject?*

Investigation showed that the greater number of those who did not continue mathematics felt that they had not gained the *sense of power* from this subject which was afforded them from other studies. Because he was not sure that each year of training "delivered the goods" to his students, Discipulus devoted his leave of absence to the investigation of high-school mathematics, seeking inspiration and clearer vision.

In training-schools for high-school teachers he met three lines of work: *why* to teach mathematics, *how* to teach it, and *what* to teach. The larger view of its place in the curriculum and the adjustment of the subject to the varying needs of different students, he found little emphasized.

Educational experts soon began to be classified by him as "teachers of mathematics" and "others." The former believed, almost to a man, that every pupil should study one year of algebra and one of geometry. The "others" were not sure that this course was justified; many, whose experience commanded consideration, condemned the current practice. "The pupil studies elementary algebra in order to acquire advanced algebra, which admits him to college algebra and thus equips

him to become a teacher of elementary algebra!" The only refutation of such a charge must come from the fact that mathematics lays the foundation for practical work, and Discipulus accordingly entered the world of applied mathematics.

His attention was immediately arrested. Many engineers of standing attached to their high-school mathematics little value except as a means of mental training. (Discipulus did not tell them that the doctrine of formal discipline was fighting hard for life.) They kept in their notebooks their few daily formulae and asked from the schools only the ability to compute accurately, to solve simple equations, and to use logarithms. Other engineers who hired large numbers of high-school students found them unable to apply mathematical common-sense to the solution of the simplest practical problem, however expert in artificial textbook problems.

Industrial schools in New York afforded a wide field of observation. Manual-training high schools employed teachers direct from the shops; night schools were thronged by those anxious to earn higher salaries. The girls' trade schools made no use of algebra except for mental training *per se*. The only mathematics required in those trades not distinctly quantitative, was simple business arithmetic and the most elementary formulae involving measurement. Attention next centered upon the training of the mechanic. Training courses for electricians, machine pattern-makers, plumbers, forgemen, etc., included algebra and geometry, yet omitted many chapters of the high-school text *in toto*. The mechanic needed the equation as a tool in handling the formulae of an engineer's handbook, but was not concerned with mathematical logic as a branch of reasoning. Also his types of factoring were fewer, his methods of handling fractions more direct and numerical.

The mathematics of the academic schools forms a logical unit which trains in abstract thinking and prepares for higher courses which few will pursue. The mathematics of the elementary technical school forms a series of simple principles which focus attention upon the equation and its applications in daily life. Two questions arise from the comparison. First,

if our high-school mathematics were to emphasize ability to solve problems rather than to develop theory, *what material should be omitted and what inserted?* The answer was most one-sided. Many were ready with new matter to introduce; none presented a reasonable scheme of what might be omitted. Second, how could the practical problem be introduced pedagogically, and would the change produce more power to think independently. The most direct answer came from the School of Science and Technology, Pratt Institute, Brooklyn, N. Y., and from Lewis Institute, Chicago.

In Pratt Institute, the traditional methods of teaching have been abandoned. The mathematics of the two-year courses, which includes the essentials of algebra, geometry, trigonometry, analytics, and calculus, is in a series of practical texts so taught as to be as constructive, creative, and individual as shop work and the product is as proudly regarded by the student. To use the work in algebra as an illustration, students begin with the formulation in applied notation of the principles and laws in common use in shop, drawing-room, and laboratory, by means of which the significance and use of algebra shall become immediately apparent. Every formula is applied in numerical computation to the end that even the weaker students shall become expert and accurate in the common arithmetical operations. Gradually, by inductive and suggestive methods, the conventional is interwoven with the concrete and practical. The work throughout the course is consciously based on the fundamental and accepted principles of education in such manner that each student writes each subject complete, with steady purpose and independence and with an increasing sense of power and useful accomplishment.

Lewis Institute in Chicago afforded the same line of comparison among a younger set of students. In one room the orthodox class in algebra employed laboratory methods, embodied the latest improvements of mathematics associations, and covered the college requirements. The student thought clearly but worked with the usual passive obedience. In the adjoining room a class of apprentices devoted alternate weeks to the shop

and to its mathematics. They studied the working of the steam engine in the laboratory, and during the next week its underlying mathematical principles. The boys worked alertly, upon their own initiative, because they understood the immediate purpose of their work.

The contrast suggested a carpenter's tool chest. In one room, every form of brace and bit, every size of chisel and screw-driver, finely tempered and perfect, hung in its proper place, ready for use in the far future. In the other, each had the simplest kit of mathematical tools, a few saws, planes, and chisels, but the shop boy kept them bright with much use upon all occasions.

One difficulty in conducting such work has been the lack of a suitable text. (It was in a far western school that Discipulus found the book which contained "psychologized elementary mechanics in terms of the child" and thus bridged the usual gap.) Another difficulty arises from the fact that the master of applied mathematics commands a higher salary than the school can afford; yet the teacher is too busy with routine work to gain first-hand knowledge in the shop.

The lesson of the technical school was manifest. It studied the law of supply and demand. Because the world needs craftsmen who can "do things," all its courses must bend toward that end. Each year's mathematical training must embody a precipitate of knowledge indispensable to the mechanic. One of the great problems of the public school it never meets: "What mathematical training should be given to those who will enter neither college nor a mechanical trade?"

From the polytechnic institute, with its freedom to unite theory and practice, Discipulus turned to the great city schools. Here it was the almost unfailing custom to prescribe one year of algebra and one of geometry to all sorts and conditions of pupils. Those who continued algebra, as an elective, for a second year, were but a small minority, and these alone could enter college. Those who studied the subject for but a single year were merely half-way toward the goal of college entrance. No residuum of knowledge for use in the practical world was

insisted upon during the first year's work. The catechism became monotonous in its repetition.

Do you think it desirable that more students should study a second year of algebra?

No.

Is it advisable that all should study one year of algebra?

Such has been our custom.

Do you in the first year give practical applications of the algebraic manipulations?

We have too little time.

Of what value, then, is this single year of algebra, since you do not give to abstract symbols a concrete meaning?

Perhaps later the student may desire to enter college.

Is the present year of work of such value as to prescribe it to the majority, for the sake of the minority who enter college?

It is not.

Why then do you emphasize the abstract at the expense of the concrete and also at the expense of the child?

Because we are influenced by the College Entrance Examinations.

Again and again, in the last analysis, the examining board was made to shoulder the blame for existing conditions. Discipulus felt moved to investigate the influence of this board upon the choice of subject-matter taught in the schools. It is possible to pass the recent college entrance examinations in elementary algebra and include but a *single question* in which mathematical symbols are given a concrete meaning. The purpose of the system is to determine "the candidate's intelligence, power, and preparedness for college work." It is obvious therefore that the examination may well serve its avowed purpose without testing the ability of the great majority to use their high-school training in daily life. The board has contributed much toward unifying the entrance standards of the colleges. It is, of necessity, a conservative body, yet it stands ready to readjust its requirements when the high school can present a scheme of applied mathematics which meets the demands of the majority better than does our present system.

If the high-school teacher finds that those of his students who want immediate power from their mathematics drift into technical classes, he must not make the College Entrance Board

his excuse. The difficulty must be squarely met. It lies in the complex nature of the remedy suggested. If the present teaching is too abstract, it can become concrete only by closer connection with practical life. One school may apply its mathematics to the law of levers, a second may emphasize horsepower and steam pressure. But suppose the examination draws its applications from projectiles? The range of data for such illustrations is world wide. The practice of sacrificing high-school freshmen upon the altar of mathematics is also world wide. The process of rescuing him by the judicious readjustment of prescribed and elective, of symbolic and applied algebra is the problem of the mathematics department. Since no one has a more intimate knowledge of the defects of the present system than has the high-school teacher, it is for him to develop the remedy.

From the technical school to the academic high school and thence to the teacher—whence come his inspiration and enlightenment? From experiments in correlating allied subjects, from daily contact with experts in modern higher mathematics, from research work in the history of its development.

The correlation of algebra, geometry, and trigonometry into a single unit may be seen in the Lincoln High School, Nebraska. During twelve years, by the most painstaking attention to detail, it has developed a science of high-school mathematics, natural, teachable, and vital. The experimental stage is passed; each week's work is a definite and clean-cut section of a well-proportioned system. Pupils are familiar with interrelated mathematical principles and their history. The talk of the classroom combines algebraic, geometric, and trigonometric concepts with the familiarity of long usage. Of importance psychologically is the fact that pupils are here required to invent problems illustrating given principles. These, when given to the class, arouse more interest than allied book problems.

At the close of the visit came the test question: "Do you expect *all* pupils to study mathematics?" The answer was significant. Work in mathematics begins in the second half of the first year, preceded by an inspirational course in general science.

Such students as have previously shown marked inability in mathematics are not urged to enter the course. The consequent freedom from dead weight makes possible sufficient joy in the work to keep the great majority studying the subject for three and a half years! Boys and girls here continue their education who would elsewhere leave school in discouragement.

A leave of absence converts a teacher into a student. Discipulus' inspiration in renewing the study of higher mathematics was due partly to working under professors with a strong grasp of a broad subject, partly to breathing again the atmosphere of the university, partly to greater intimacy with the work of today's mathematicians. This latter is discouraging because the various developments of modern mathematics are so highly specialized. Each has its own new set of symbols and brings the student, at the end of the year, but to the threshold of new knowledge. Courses, or series of articles surveying the mathematical contributions of the twentieth century "in words of one syllable" would be worth their weight in gold.

Throughout the year, work in the history of mathematics afforded unending pleasure. The riches of the mathematical library and exhibit at Teachers College, manuscripts gathered from all quarters of the world, first editions of mathematical classics, translations, fresh from the press, of the latest mathematical discovery, the mysticism of number, its puzzles and games—the number systems of the ages, the development of mechanical computation—the ancient calendars, astronomical instruments, weights, and measures—the book-lover longed for a lifetime to enjoy such abundance of riches. The student searched for new light on ancient modes of thought, the teacher anticipated the pleasure of illuminating his class work with historical sidelights.

Unwavering was the scholarly help and kindly guidance of the master of the library, unwavering his dictum. "As is the development of an idea in the history of the world, so should be its method of presentation to the class." Nowhere could the inconsistency of our present practice be more startlingly revealed. The ancients developed the special case of a mathe-

matical principle centuries before the general. Today the general principle begins the high-school chapter and precedes the special cases. Slowly and with labor the philosophers of old taught to the chosen few. They demanded a mature man's judgment to grasp the subtleties of logic. The moderns cover in two years a system of algebra whose development occupied two millenniums. No longer is it chosen by the elect, it is *prescribed* wholesale for the high-school freshmen!

Discipulus returned to think over the year's work. Despite much studying and visiting that have been left undone and some that has been done amiss, he has faith that the following conclusions are true:

I. *Concerning the place of mathematics in the curriculum.*—The renaissance in high-school mathematics concerns itself with what to teach and how to teach it. The question of the *advisability* of the present training for all sorts and conditions of students is still dormant.

II. *Concerning standards of judgment.*—The first and second years of mathematics need to be brought to judgment. Poor work in first-year Latin is revealed in the second year. Poor work in first-year English may be discovered by the parent. Poor work in mathematics frequently passes unremedied because the student rarely continues his subject. The value of his work is not "weighed in the balance."

III. *Concerning the focus of subject-matter.*—The first two years of secondary mathematics suffer from a lack of perspective. If each year's work were divided into *essentials* and *non-essentials,* the work of the classroom would be more efficient.

IV. *Concerning a content of mathematics as knowledge.*—If a certain precipitate of knowledge were considered essential throughout his course, the student of mathematics would refute the criticism that he does not "know things." Were this content selected from the viewpoint of the mechanic rather than of the logician, the student would be better able to apply principles in practice.

V. *Concerning the practical problem.*—The present status of the practical problem is ambiguous. Attention is chiefly confined to collecting and developing problems which frequently involve unduly difficult principles of physics. These may be replaced to advantage by simpler material from polytechnic shops and engineer's handbooks.

VI. *Concerning who should study applied mathematics.*—No one doubts that the introduction of the applied problem makes mathematics more practical. Does it do so for the girl of literary tastes as well as for the incipi-

ent engineer? There is need to develop the pedagogy and psychology of this question.

VII. *Concerning "team-work" in the faculty*—Much poor teaching passes unremedied and much good work unrecognized for lack of expert supervision. The practice of overworking heads of departments and providing them with poorly trained assistants is pernicious.

VIII. *Concerning the future.*—Progress can be made only by eliminating non-essentials from the present courses. We need experiments to discover what topics may be omitted to make room for the riches of applied mathematics.